HIGH-ENERGY PART OF ION DISTRIBUTION FUNCTION IN A MULTIPLE-MIRROR MAGNETIC TRAP

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The considerable reduction in the thermal conductivity and in the rate of longitudinal expansion of plasma in multiple-mirror magnetic traps [1] makes such systems very promising from the point of view of thermally insulating a dense plasma along the magnetic field in a thermonuclear reactor. The behavior of the plasma in multiple-mirror traps can be qualitatively described with the aid of equations of gasdynamic type, derived in [2, 3]. The validity of these equations has been confirmed experimentally on setups with an alkali plasma [4, 5].

A pressing problem in devices of the next generation with a deuterium or deuterium–tritium plasma is the measurement of the output of neutrons from the plasma (primarily for diagnostic purposes). In this connection the question arises of finding the relationship between the neutron output and the macroscopic parameters of the plasma.

In installations of the type in question, this problem has the following distinctive feature. The system of gasdynamic equations [1-3] correctly describes the distribution function of particles whose mean free path λ is small compared with the length of the installation L. In cases of practical interest this condition is satisfied for the main mass of the particles of the plasma; however, as is well known (see [6], for example), the main contribution to the neutron output at not too high plasma temperatures ($T \leqslant 40$-50 keV) comes from the high-energy "tails" of the distribution functions. Now the mean free path increases rapidly with temperature; accordingly, in order to find the neutron output, it may happen that we require to know the distribution function at energies ε for which $\lambda(\varepsilon) > L$ and where, consequently, the results obtained in [1-3] cannot be utilized.

The aim of the work reported here* was to find the distribution functions in the range of energies for which $\lambda(\varepsilon) > L$.

1. Asymptotic Behavior of Distribution Function in Deuterium or Deuterium – Plasma

Particles with $\lambda(\varepsilon)/k \ll L$ leave the installation in a diffusion manner, experiencing many collisions. For these particles the distribution function is close to Maxwellian. In the range of energies for which $\lambda(\varepsilon)/k \gg L$, the efflux of particles into the loss cone of the installation begins to play an important role in the formation of the distribution function in addition to Coulomb collisions. As is well known, Coulomb collisions result, first of all, in an influx of particles into the high-energy region due to the diffusion of the particles in velocity space and, secondly, in a return flow of particles to the low-energy region due to the force of dynamic friction that a particle moving in the medium experiences. Under steady-state conditions in an unbounded plasma without any losses, the equality between these two fluxes maintains in a dynamic manner the Maxwellian distribution function of the particles. The additional efflux of particles into the loss cone in multiple-mirror traps results, evidently, in the distortion of this function. We obtain below the asymptotic form of the distribution function for deuterium and tritium nuclei in the region of energies for which $\lambda(\varepsilon)/k > L$.

According to [1-3] a plasma is confined most effectively when the following two conditions are satisfied: 1) The length of an individual mirror cell l is of order $\lambda(T)/k$; 2) the length of the mirror (i.e., the length of the

*In cases when the mirror ratio $k \equiv H_{max}/H_{min}$ is large compared with unity, the criterion of applicability of the gasdynamic approach is that $\lambda(\varepsilon)/k < L$ [and not $\lambda(\varepsilon) < L$], which will be taken into account below.

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region where the magnetic field varies from H_{min} to H_{max}) is much less than l^*. We assume below that these conditions are satisfied, so that in particular the mirror length can be neglected compared with the cell length.

The density n and the temperature T (which characterize the main mass of plasma particles) can be regarded as known functions of the coordinates and time: They can be found within the framework of the gas-dynamic equations of [2, 3]. Since the fall in electrostatic potential over the length of an individual mirror cell does not exceed $(T/e)(l/L)$, the effect of the electric field on the motion of particles with energy $\varepsilon \gg T$ can be neglected. It is meaningful to restrict the discussion only to that region of energies of particles with $\lambda(\varepsilon)/k > L$ in which the distribution functions at each moment of time manage to adjust themselves in n and T. For particles with greater energies, the form of the distribution functions is determined by prescribing them at the initial moment of time and by the rate of the subsequent decay of this state. Installations in which this group of particles makes the main contribution to the neutron output are of little interest from the practical point of view. In this manner, the time derivative of the distribution function can be dropped from the kinetic equations; in the final result the dependence on time enters only parametrically, through the functions n(t) and T(t).

Under the above conditions (steady-state problem, smallness of electric field, uniform magnetic field in space between mirrors), the following equation can be written for the distribution function of particles of species a:

$$\sum_b St_{ab}\{f_a, f_b\} = 0,$$

where St_{ab} is the collision integral between particles of species a and species b [7]; f_a and f_b are the respective distribution functions. Remembering that the ion velocity in the considered range of energies $T \ll \varepsilon \ll T(m_a/m_e)$ (where m_a is the mass of an ion of species a; m_e is the electron mass), the collision integral can be brought to the following simple form:

$$\sum_b St_{ab} = \frac{2\pi\Lambda e_a^2}{m_a^2 v_{Ta}^3}\sum_b{}' e_b^2 n_b \frac{m_a}{m_b}\left\{\frac{1}{v^2}\frac{\partial}{\partial v}\left(2f_a + \frac{1}{v}\frac{\partial f_a}{\partial v}\right) + \right.$$

$$\left. + \frac{\delta_1}{v^2}\frac{\partial}{\partial v}\left(2v^3 f_a + v^2\frac{\partial f_a}{\partial v}\right) + \frac{\delta_2}{v^3}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial f_a}{\partial\theta}\right\} = 0; \tag{1.1}$$

$$\delta_1 = \frac{4}{3\sqrt{\pi}}\left(\frac{m_e}{m_a}\right)^{1/2}\frac{e^2 n_e}{\sum_b{}' e_b^2 n_b \frac{m_a}{m_b}}, \quad \delta_2 = \sum_b{}' e_b^2 n_b \Big/ \sum_b{}' e_b^2 n_b \frac{m_a}{m_b}, \tag{1.2}$$

where v is measured in units of $v_{Ta} \equiv \sqrt{2T/m_a}$, and the primes at the summation signs indicate that the summation is carried out only over ions (electrons are excluded from the summation). The second term in the curly brackets describes the collisions of ions with electrons (the plasma is assumed isothermal, $T_e = T_i = T$, which is valid for sufficiently long plasma bunches [2]).

The boundary conditions for Eq. (1.1) have the form

$$f_a|_{\theta=\theta_0} = 0; \quad f_a(v)|_{v=v_0} = f_{a0}; \quad f_a(v)|_{v\to\infty} \to 0.$$

The first of these corresponds to the reduction to zero of the distribution functions of fast ions on the boundary of the loss cone $\theta = \theta_0 \equiv \arcsin\sqrt{H_{max}/H_{min}}$, in the second the quantity v_0 is chosen so that $\lambda(v_0)/k \sim L$ (this automatically means that $v_0 \gg 1$); the significance of the third is obvious.

At the point $v = v_0$ a transition occurs from the Maxwellian distribution function obtaining in the region $v < v_0$ to a distribution function that diminishes more rapidly in the region $v > v_0$. A precise determination of $f(v_0)$ would involve solving the problem in the transition region $v \sim v_0$ with all the attendant mathematical difficulties. However, it is clear from the outset that at $v = v_0$ the distribution function is not too different from Maxwellian; accordingly, we shall assume $f(v_0)$ to be known and equal to the value of the Maxwellian distribution function at this point. As a result of this approximation, the obtained solution will be valid, correct to a certain numerical multiplier of the order of unity.

Equation (1.1) permits separation of variables, and we represent its solution in the form

$$f_a = \sum_{m=0}^{\infty} A_m R_m(\mu_m, \theta) Q_m(v) \tag{1.3}$$

$$R_m(\mu_m, \theta) = F\left(\frac{1-\sqrt{4\mu_m+1}}{4}; \frac{1+\sqrt{4\mu_m+1}}{4}; \frac{1}{2}; \cos^2\theta\right),$$

*The use of "point" mirrors of this sort is advantageous in that only small amounts of magnetic energy are required to produce them.

where F is the hypergeometric function, defined as in [8]. The eigenvalues μ_m are determined from the boundary condition $R_m(\mu_m, \theta_0) = 0$, and the coefficients of the expansion have the form

$$A_m = \int_{\theta_0}^{\pi-\theta_0} f_{a0} R_m \sin\theta d\theta \Big/ \int_{\theta_0}^{\pi-\theta_0} R_m^2 \sin\theta d\theta.$$

At large mirror ratios $k \gg 1$ the functions $R_m(\mu_m, \theta)$ are little different from the Legendre polynomials of degree 2m for all θ except for narrow regions at the points $\theta = 0, \pi$, while the μ_m are close to the values $2m(2m+1)$ for $m = 1, 2, 3, \ldots$. For $m = 0$ the first term of the expansion of μ_0 with respect to k has the form

$$\mu_0 = 2/\ln k, \quad R_0(\mu_0, \theta) = 1 - \ln \sin^2\theta / \ln k. \tag{1.4}$$

Numerically, expression (1.4) is a good approximation of μ_0 up to values of $k \gtrsim 2$. For example, the value $\mu_0 = 2$ corresponds to the solution $R_0(\mu_0, \theta)$ given by the associated Legendre polynomial of the first degree:

$$R_0(\mu_0, \theta) = 1 - \frac{1}{2}\cos\theta \ln\frac{1+\cos\theta}{1-\cos\theta};$$

here k, found from the condition $R_0(\mu_0, \theta_0) = 0$, equals 2.8, which is in good agreement with (1.4).

We note that the eigenvalues μ_m decrease monotonically with increasing k. It is also known that the limiting value of μ_m as $k \to \infty$ is $2m(2m+1)$, which means that for arbitrary k the inequality $\mu_m > 2m(2m+1)$ holds for all m [this relationship is utilized below when estimating the magnitudes of terms with different m in expansion (1.3)].

The functions $Q_m(v)$ satisfy the equation

$$\frac{d}{dv}\left(\frac{1}{v}\frac{dQ_m}{dv} + 2Q_m\right) + \delta_1 \frac{d}{dv}\left(v^2\frac{dQ_m}{dv} + 2v^3 Q_m\right) - \frac{\mu_m \delta_2}{v} Q_m = 0 \tag{1.5}$$

with boundary conditions $Q_m(v_0) = 1$, $Q_m(\infty) = 0$.

The last term in (1.5) corresponds to the leakage of particles into the loss cone. Without this term the solution of (1.5) satisfying the boundary conditions would be the Maxwellian function. When leakage is taken into account the distribution function will, clearly, fall off more rapidly than Maxwellian, i.e., $Q_m(v)$ has the form

$$Q_m = q_m(v)\, e^{-v^2+v_0^2}, \tag{1.6}$$

where $q_m(v_0) = 1$; $dq_m/dv < 0$. Inserting the solution in the form (1.6) into (1.5) gives

$$\frac{1}{v}\frac{d^2q_m}{dv^2} - 2\frac{1+\frac{1}{2v^2} - \delta_1 v + \delta_1 v^3}{1+\delta_1 v^3}\frac{dq_m}{dv} - \frac{\mu_m\delta_2}{v(1+\delta_1 v^3)} q_m = 0.$$

The condition $v \gg 1$ means that in the coefficient of dq_m/dv the term $1/2v^2$ can be neglected compared with unity and the term $\delta_1 v$ neglected compared with $\delta_1 v^3$, which gives

$$\frac{1}{v}\frac{d^2q_m}{dv^2} - 2\frac{dq_m}{dv} - \frac{\mu_m\delta_2}{v(1+\delta_1 v^3)} q_m = 0. \tag{1.7}$$

For a deuterium or deuterium–tritium plasma $\delta_1 \sim 10^{-2}$, $\delta_2 \sim 1$. Thus, for mirror ratios not too close to unity ($k - 1 \gtrsim 1$), the last term in Eq. (1.7) contains for the first few m the small multiplier

$$\eta_m(v) = \mu_m\delta_2 / v(1 + \delta_1 v^3) \qquad (v > v_0 \gg 1). \tag{1.8}$$

If this multiplier were equal to zero, one of the two linearly independent solutions of (1.7) would be a constant; the effect of the finiteness of η_m is to make this solution slowly decreasing. It is readily shown that this solution can be sought neglecting the first term in (1.7), i.e., writing (1.7) in the form

$$\frac{dq_m}{dv} = -\frac{\mu_m\delta_2}{2v(1+\delta_1 v^3)} q_m,$$

whence, utilizing the boundary conditions $q_m(v_0) = 1$, we obtain

$$q_m(v) = \left(\frac{v_0^3}{1+\delta_1 v_0^3}\right)^{\frac{\mu_m\delta_2}{6}} \left(\frac{1+\delta_1 v^3}{v^3}\right)^{\frac{\mu_m\delta_2}{6}}. \tag{1.9}$$

The second linearly independent solution of Eq. (1.7) is rapidly (exponentially) increasing and must be discarded.

Since $\mu_m > 2m(2m+1)$ it follows that $q_m(v)$ decreases rapidly with increasing m. Accordingly, when calculating the neutron output, only the first one or two terms need be retained in (1.3).

In the region $v \gg v_0$ the main contribution to f_a clearly comes from the term of expansion (1.3) with $m=0$. As $v \to \infty$, $q_0(v)$ approaches the limit $\left(\frac{\delta_1 v_0^3}{1+\delta_1 v_0^3}\right)^{\frac{\mu_0 \delta_2}{6}}$. Numerically, this quantity differs appreciably from unity only for relatively small values of the mirror ratio $k \sim 3$ [for $k=2.8$, $\mu_0=2$, taking $v_0=1$, we obtain $q_0(v)_{v\to\infty} \to 1/5$; since the output of neutrons from the plasma is proportional to the square of f_a this sort of depletion of f_a in the region $v > v_0$ can lead to an appreciable drop in the neutron output]. For $k \gg 3$, f_a is approximated well by the Maxwellian function almost everywhere in velocity space except for the loss cone.

2. Asymptotic Behavior of Distribution Functions of Deuterium and Tritium Nuclei in Plasma with Impurities in the Form of Multiply Charged Ions

The use of multiply charged ions to improve plasma confinement in multiple-mirror traps is suggested in [9]. It is well known that the scattering cross section for particles over angles increases in proportion to the squares of the charges of the particles, with the result that even small admixtures of multiply charged ions $n_a/Z^2 \ll n_Z \ll n_a/Z$ (Z is the multiplicity of the charge of the ions; n_Z is their concentration) can considerably reduce the mean free path of deuterium and tritium particles and slow down the rate of diffusion leakage of plasma along the device. However, the fact that f_a becomes isotropic more rapidly has a marked effect on the form of the distribution functions of D and T in multiple-mirror traps with $\lambda(\varepsilon)/k > L$.

Indeed, the increase in the rate of scattering of D and T particles over angle leads, in the region $\lambda(\varepsilon)/k > L$, to a more rapid leakage of particles into the loss cone compared with the case of a D–T plasma without impurities. Since, on the other hand, the presence in the plasma of an admixture of multiply charged ions with $n_Z \ll n_a/Z$ has no significant effect on the process of diffusion of the particles over energy in velocity space, this increase in the leakage of particles clearly leads to a more rapid fall of f_a with increasing v.

Formally, the presence in the plasma of multiply charged ion impurities with $n_a/Z^2 \ll n_Z \ll n_a/Z$ corresponds in the equations to a value of $\delta_2 \gg 1$ [see (1.2)]. Of main interest when estimating f_a is the behavior of $q_0(v)$. Approximation (1.9) for $q_0(v)$ is valid so long as the inequality $\eta_0(v_0) \ll 1$, where $\eta_0(v)$ is given by formula (1.8), holds. For a plasma with large impurity concentrations n_Z this condition is violated, and in a certain range of velocities $v > v_0$ we have that $\eta_0(v) \gg 1$. In order to find the asymptotic behavior of $q_0(v)$ in this case, we insert $q_0(v)$ in the form $q_0(v) = \exp[-h_0(v)]$ in (1.7). Retaining only terms quadratic in h(v), we obtain

$$(h'(v))^2 = \mu_0 \delta_2 \frac{1}{1+\delta_1 v^3},$$

$$h(v) = \sqrt{\mu_0 \delta_2} \int_{v_0}^{v} \frac{dv}{(1+\delta_1 v^3)^{1/2}}.$$

In the region $\eta_0(v) \ll 1$, $q_0(v)$ coincides with (1.9) correct to a numerical factor.

In this manner, in a plasma with multiply charged ion impurities $n_Z \gg n_a/Z^2$ for installations with not too large mirror ratios k (such that $\mu_0\delta_2/6 = \delta_2/3 \ln k \gtrsim 1$), a strong depletion of f_a may be observed compared with the Maxwellian function at large particle energies $\varepsilon \gg T$ and a sharp fall in neutron output may occur.

Let us estimate the velocity from which the distribution function starts, in the transition region $\lambda(\varepsilon)/k \sim L$, to drop compared with Maxwellian. The rate at which f_a falls off due to diffusion leakage of plasma along the coordinate is given by

$$\partial f_a/\partial t \simeq -D \partial^2 f_a/\partial s^2$$

with a diffusion coefficient $D = (\lambda(a)/k^2)v$, while the rate at which it is restored due to the diffusion of particles over energy in velocity space is given by

$$\frac{\partial f_a}{\partial t} \gtrsim \frac{1}{\delta_2 \lambda(T)} \frac{v_{Ta}^6}{v^3} \frac{\partial^2 f_{\mu a}}{\partial v^2}.$$

Comparing the first and second rates gives $v_1 \simeq \delta_2^{-1/6}(Lk/\lambda(T))^{1/3}v_{Ta}$. The fact that v_1 must clearly satisfy the inequality $\lambda(v_1)/k < L$ implies that f_a is close to the Maxwellian function in almost the entire region $\lambda(\varepsilon)/k \leqslant L$ for concentrations $n_Z < (Lk/\lambda(T))^{1/2}n_a/Z^2$.

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NUMERICAL EVALUATION OF DIODE GAP BRIDGING BY IONS IN A PLANAR DIODE

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To obtain powerful electron beams the so-called planar diodes, in which the cathode as well as the anode are disks of radius R exceeding considerably the gap a between the electrodes, are often used. If the self-magnetic field of the beam can be ignored (for example, when the diode is in a strong external magnetic field), then the motion of the electrons in the diode is one-dimensional. The problem of determining the stationary current passing through such diode in the nonrelativistic case is easily solved, the corresponding dependence being given by the well-known "3/2 rule." This solution can be extended to relativistic potentials [1]; moreover, the ion emission from the anode can also be included [2]. It is assumed again that the diode current is time-independent. In this article results of numerical computations of the nonstationary electron-diode operation in a state in which the diode gap is filled by ions emitted from the anode are given. The case of ion emission of one type, as well as the cases in which ions are emitted with different masses, is studied.

1. Formulation of the Problem

The anode plasma which arises as a result of diode operation is a source of ions emitted into the diode gap. The distinctive time scale τ of the problem is the duration of ion transit between the electrodes a, $\tau \sim a(M/e\varphi_0)^{1/2}$, where φ_0 is the potential difference between the cathode and the anode; M is the ion mass (ions are regarded as singly charged). It is of interest to analyze the times $t \leqslant \tau$ when the problem is essentially nonstationary (the time t is counted from the start of the ion emission). For $t \gg \tau$ the diode operates in a stationary manner with an incoming flow of electrons and ions; in the latter case the results of [2] become applicable. It is assumed that in the period of time which is short compared with τ a sufficiently dense plasma

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